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**Research article**

# Multiobjective Mathematical Programming Approach to Minimize Volume of Solid Waste at Waste Collection Centers in Municipalities

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## ABSTRACT

Solid waste is a serious environmental problem in urban centers of both developed and developing countries, due to phenomenal growth in population. Evacuation of municipal solid waste (MSW) is very expensive as it involves huge amount of budgetary allocation by management authorities. In this paper, a Multi-objective Mathematical Programming model is developed to address the conflicting objectives of minimization of the volume of waste in various collection centers in a municipality, considering comparable importance of the collection centers and associated cost. An application of the model is demonstrated using data collected from Abuja Municipal Solid Waste Management Agency (Abuja Environmental protection Board –AEPB), waste management companies, waste recycling vendors and other concerned stakeholders in Abuja, Nigeria. The model was constructed on a Spreadsheet and solved using Microsoft Excel Solver 14.0. The study revealed that all the volumes of wastes in the collection centers in the municipality can be evacuated with 67% of the amount allocated daily. The remainder 33 % of total amount allocated daily was discovered to have been lost to contracted companies after the evacuation. 

**Keywords:** Multi-objective mathematical programming, Municipal solid waste, Waste evacuation, Cost minimization, Waste collection centers.

## INTRODUCTION

Any unwanted discarded materials that is not a liquid or gas, which normally comes from households, commercial and institutional establishments, markets, street sweeping, demolition and construction of buildings, etc., is referred to as Municipal Solid Waste (MSW) [18]. For example, newspaper waste, junk mails, meal scraps, pieces of bread, thrown away batteries and tyres, waste rice, racked leaves, dust, grass clippings, broken furniture, abandoned materials, animal manure, street sweepings and any solid produced by humans. Waste material is produced as a result of human activity. The volume of this material is increasing readily due to increase in human population and standards of living [14].

Urban centers are faced with increased problem of solid waste due to population growth, industrialization, urbanization and economic growth. A trend of significant increase in MSW generation has been recorded worldwide [22]. Waste generation has been observed to increase annually in proportion to the rise in population and Urbanization [19]. Management of MSW has become a serious problem in most urban centers of the world, as a result of continues increase in volume of wastes.

Environmental and social issues emerge as people become increasingly concerned about the risks associated with these wastes not evacuated for a long period of time [11 and 21]. A visit to most African cities today will reveal aspects of the waste-management problem such as heaps of uncontrolled garbage, roadsides littered with refuse, streams blocked with junks, and disposal sites which constitutes health hazard to residential areas. This is partly caused by lack of investment in modern technology and well functional MSW management system coupled with rapid population growth and urbanization [23].

Accumulated solid waste, if not properly managed has serious environmental, social and health problems. [7] attributed flooding in Lagos to clogging of drainage channels by dumped solid wastes. Improper disposal of solid wastes pollutes all the vital components of the living environment (i.e., air, land and water). The release of gaseous toxic substances and emission of toxic pollutants to surrounding environment during waste management, expose communities and individuals to serious health risk, such as, damage to the immune system, neuron, reproductive organs, developmental organs, and respiratory system, cancer, etc [6 and16].

Solid waste management (SWM) involves activities associated with generation, storage and collection, transfer and transport, treatment and disposal of solid wastes. The management of MSW requires proper infrastructure, maintenance and upgrade for all activities. These has become increasingly expensive and complex due to the continuous and unplanned growth of urban centers. The difficulties in providing the desired level of public service in the urban centers are often attributed to the poor financial status of managing municipal corporations.

Various researchers have undertaken to study solid waste management, but most of the studies are usually case study of particular communities on facility site selection, facility capacity expansion, facility operation, vehicle routing, manpower assignment, over-all system operation, system scheduling and waste flow [5, 9, 10, 15, 23, 20 and 3]. For example, [8] developed a linear programming model to integrate different methods of waste management in Bangalore, India. [5] developed a multi-criteria facility location model for municipal solid waste management in North Greece. The model tends to prove optimal location and allocation decision of waste facilities. [15] developed a multi-objective integer goal programming model for analyzing problems involving planning and design of regional hazardous waste management system. The model was developed to select treatment and disposal facilities along with allocation of hazardous wastes to various facilities in waste management system. [20] developed multi-objective optimization of solid waste flows. Reference point methodology was used within an iterative procedure to model a multi-objective decision process for sustainable MSW management. The decision process was aimed at optimizing the flow of solid waste sent to landfills, recycling and different types of treatment plants, as well as determining the size of such plants. However, studies on MSW management based on collection centers have not been adequately documented in the literature.

Collection centers have their peculiarities in terms of types of waste generated, volume of waste generated, population density, water ways and road network etc. Since management authorities/agencies are faced with limited financial resources for waste management, there is the need to prioritize waste management at collection centers so as to minimize cost associated with environmental, social and health problems.

In this paper, a multi-objective mathematical model to minimize the cost of evacuation of volume of waste at various collection centers in municipalities based on some identified criteria is developed.

## METHODOLOGY

Multi-objective Programming algorithm was considered in developing the proposed model. The algorithm is one of the Multi-criteria Decision Making (MCDM) technique in which decision problems with several conflicting criteria are considered [20 and 12]. In this study, MSW management planning was structured as a Multi-objective Decision Making ((MODM) problem with several conflicting criteria. MODM can be divided into three parts namely; preference, interactive and non-preference type (lexicographic, multi-attribute utility and unknown utility) [12].

In this study, Interactive Multi-objective Programming approach was adopted. The method involves determining relative importance of the attributes and aggregating them into some kind of overall objective. The optimization problem is solved to generate the optimal solution for a given set of attributes. The method weighs the objectives to obtain Pareto optimal solutions. That is, each objective incorporates user supplied weights based on their relative importance and sum up to give a single objective to be minimized.

Given a multi-objective optimization with $k$ objectives, the weighted problem is as follows:

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$$\left.\begin{aligned} &\text{Min: } Z(X) = \sum_{i=1}^{k} w_i f_i(x) \\ &\textit{Subject to}: \\ &\qquad x \in S \\ &\qquad \sum_{i=1}^{k} w_i = 1 \\ &\qquad w_i \geq 0; \quad i = 1, 2, \ldots, k \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots \quad (1)$$

where, $Z(X)$ denotes the objective functions, $X$ represents the sets of decision variables, and $S$ represents the sets of constraints. Any set of nonnegative weights $w_i$ may be used in (1). However, without loss of generality, we can normalize all weights such that $\sum_{i=1}^{k} w_i = 1$. The optimization problem in (1) is a single-objective optimization problem that can be solved by existing methods such as, Graphical method (applied to two or three variables problems), Sequential Goal Programming method, Multi-phase simplex method [12]. The conceptual diagram of the proposed model is given in Fig.1.

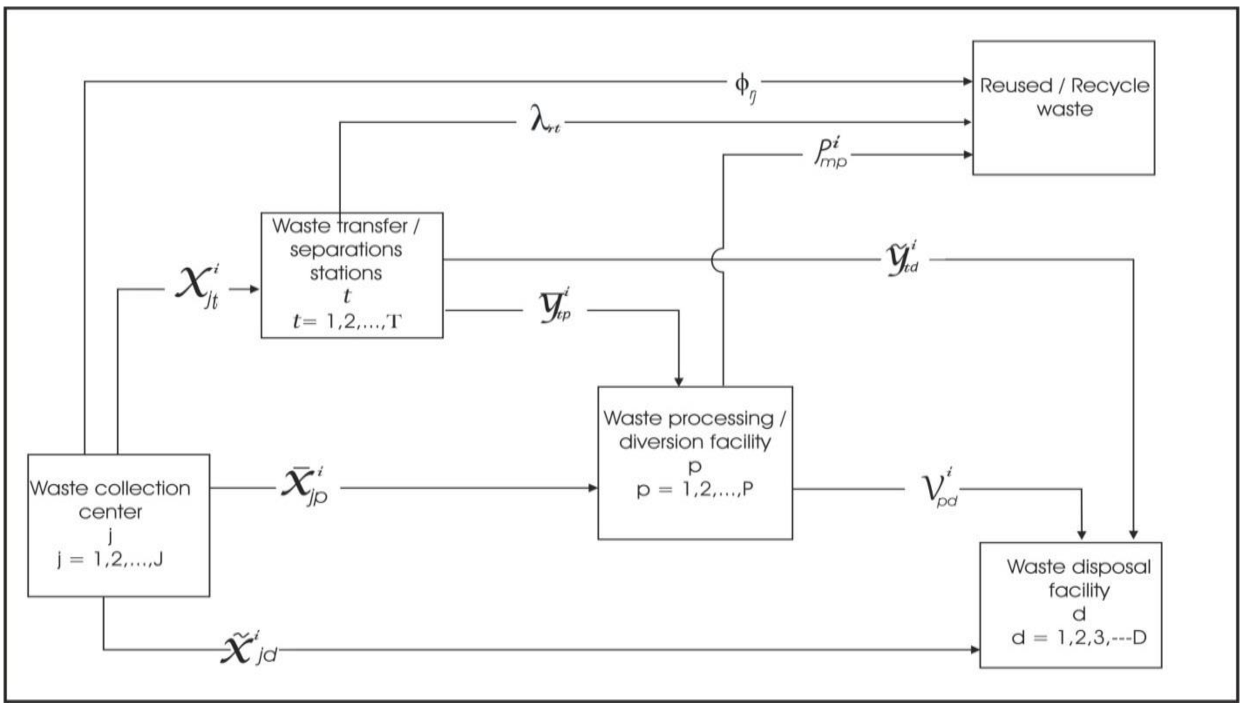


**Fig. 1:** Conceptual Model of MSW Management Process.

Figure 1 shows representation of network of the volume of waste flow from waste collection points $j \ (j = 1, 2, 3, ..., \mathrm{J})$ to final disposal facilities $d \ (d = 1, 2, 3, ..., \mathrm{D})$ in the MSW management system. In between are waste transfer stations $t \ (t = 1, 2, 3, ..., \mathrm{T})$

and various waste processing/diversion facilities $p \quad (p = 1,\ 2, 3, ..., \mathrm{P})$. The flow of volume of waste of type $i$ from waste collection point $j$ to a particular transfer station $t$, processing/diversion facility $p$, and disposal facility $d$ (residual) is represented by $x_{jt}^{i}$ , $\bar{x}_{jp}^{i}$ and $\tilde{x}_{jd}^{i}$ respectively. Flow of volumes of wastes of type $i$ from a particular transfer station $t$ to a particular processing/diversion facility $p$ and disposal facility $d$ (residual) is represented by $\bar{y}_{tp}^{i}$ and $\tilde{y}_{td}^{i}$. respectively. Flow of volumes of residues from processing/diversion facility $p$ to disposal facility $d$, when waste type $i$ is processed by the processing/diversion facility is presented by $\tilde{v}_{pd}^{i}$. Flow of fractional volumes of recovered material $r$ to the market from collection center $j$ is represented by $\phi_{rj}$. Flow of fractional volumes of material $r$ to the market from transfer station $t$ when waste type $i$ is processed is represented by $\lambda_{rt}^{i}$. Flow of fractional volumes of recovered material $m$ (reused/recycle materials, compost material, refuse derived fuel etc.) to the market from processing/diversion facility $p$ when waste type $i$ is processed is represented by $\rho_{mp}^{i}$.

## Data Collection

Data was collected from Abuja Environmental Protection Board. It includes the waste collection centers, amount of waste generated per day at the centers, percentage of waste type and composition, waste collection costs per ton, waste transportation cost per ton, waste processing cost per ton, revenue generated from reused/recycled materials per ton. Data on different types of waste management facilities (transfer station, processing/treatment facilities, disposal facilities and their respective capacities in tons, cost of waste handling/processing per ton, efficiency of the facilities) were also collected. These data was collected to assess the model. Personal interview, structured questionnaire and secondary data from the records of the organization were used to obtain the relevant data.

## MODEL FORMULATION

The primary decision variables of the solid waste management system correspond to the flow of volumes of waste materials. That is, the amount of solid waste of type $i$ moved from collection point $j$ to various facilities $t, p, d$, denoted as $x_{jt}^{i}$, $\bar{x}_{jp}^{i}$, $\tilde{x}_{jd}^{i}$ , amount of waste moved from transfer stations to processing/diversion and disposal facilities is denoted as $\bar{y}_{sp}^{i}$ , $\tilde{y}_{sd}^{i}$, while the amount of unrecovered material moved from processing/diversion facility $p$ to disposal facility $d$ when waste type $i$ is processed is denoted by $v_{pd}^{i}$

**Assumptions:**

i) Wastes generated in each community are collected at designated collection centers.

ii) Wastes generated are separated into type/category at the collection points or at transfer stations.

iii) Waste type or categories at collection centers are only moved to transfer station facility, processing/diversion facility or disposal site.

**Volume of Wastes in the Various SWM Facilities:**

- *Volumes of Waste generated at collection centers to be moved to the various facilities:*

  The volumes of wastes of type $i$ to be moved from various collection center $j$, to various facilities in the SWM system:

$$G_j\left(x,\bar{x},\tilde{x}\right)=\sum_{i\in I}\sum_{t\in T}x_{jt}^{i}\tau_{jt}^{i}+\sum_{i\in I}\sum_{p\in P}\bar{x}_{jp}^{i}\bar{\tau}_{jp}^{i}+\sum_{i\in I}\sum_{d\in D}\tilde{x}_{jd}^{i}\tilde{\tau}_{jd}^{i}\ , \qquad \forall j \tag{2}$$

  $\tau_{jt}^{i},\bar{\tau}_{jp}^{i},\tilde{\tau}_{jd}^{i}$ are (0,1) binary variables indicating whether waste type $i$ from collection point $j$ can be moved to transfer station $t$, processing/diversion facility $p$, disposal facility $d$ or not.

- *Volume of Waste at transfer stations to be moved to various facilities in SWM system:*

$$H_t\left(x\right)=\sum_{i\in I}\sum_{j\in J}\left[x_{jt}^{i}\tau_{jt}^{i}\left(1-\sum_{r}\lambda_{rt}\phi_r\right)\right], \qquad \forall t \tag{3}$$

  $\lambda_{rt}$ is the percentage of reused/recycled material $r$ recovered at transfer station $t$.

- *Volume of waste at processing/diversion facilities to be moved to various facilities in the SWM system:*

$$N_p\left(\bar{x},\bar{y}\right)=\sum_{i\in I}\sum_{j\in J}\left[\bar{x}_{jp}^{i}\bar{\tau}_{jp}^{i}\left(1-\sum_{m}\rho_{mp}\right)\right]+\sum_{i\in I}\sum_{t\in T}\left[\bar{y}_{tp}^{i}\bar{h}_{tp}^{i}\left(1-\sum_{m}\rho_{mp}\right)\right], \qquad \forall p \tag{4}$$

  $\rho_{mp}$ is the fraction of recovered material $m$ at processing facility $p$ .

**Waste Management Cost**

Solid waste management cost was categorized into investment and operation costs. The operation costs are of two types, namely, waste processing and facility maintenance cost and transportation cost. Both are linear in the amount of waste handled. Investment expenditures include facility construction/ facility expansion.

- *Transportation Cost:*

(i) Cost of transportation of the waste type $i$ from various collection point $j$ to transfer stations, processing/diversion facilities, and disposal facilities, is given as:

$$F_1(x,\bar{x},\tilde{x})=\sum_{i\in I}\sum_{j\in J}\sum_{t\in T}q_{jt}^{i}x_{jt}^{i}\tau_{jt}^{i}+\sum_{i\in I}\sum_{j\in J}\sum_{p\in P}\bar{q}_{jp}^{i}\bar{x}_{jp}^{i}\bar{\tau}_{jp}^{i}+\sum_{i\in I}\sum_{j\in J}\sum_{d\in D}\tilde{q}_{jd}^{i}\tilde{x}_{jd}^{i}\tilde{\tau}_{jd}^{i} \tag{5}$$

where $q_{jt}^{i},\bar{q}_{jp}^{i},\tilde{q}_{jd}^{i}$ , are cost of transportation of unit waste of type from collection point $j$ to transfer st ation $t$, processing/diversion facility $p$ and disposal facility $d$ .

(ii) Cost of transportation of waste type $i$ from transfer stations to processing/diversion facilities and disposal facilities is given as:

$$F_2\left(\bar{y},\tilde{y}\right)=\sum_{i\in I}\sum_{t\in T}\sum_{p\in P}\bar{b}_{tp}^{i}\bar{y}_{tp}^{i}\bar{h}_{tp}^{i}+\sum_{i\in J}\sum_{t\in T}\sum_{d\in D}\tilde{b}_{td}^{i}\tilde{y}_{td}^{i}\tilde{h}_{td}^{i} \tag{6}$$

$\bar{b}^i_{sp}, \tilde{b}^i_{sd}$ are cost of transportation of unit waste of type $i$ from transfer station $t$ to processing/diversion facility $p$ and disposal facility $d$, $\bar{h}^i_{tp}, \tilde{h}^i_{td}$ are (0,1) binary variable indicating whether waste type $i$ from transfer station $t$ can be shifted to processing/diversion facility $p$ or disposal facility $d$.

(iii) Cost of transportation of residue from various processing/diversion facilities to disposal facilities when waste type $i$ is processed is given as:

$$F_3(v) = \sum_{i \in I} \sum_{p \in P} \sum_{d \in D} g^i_{pd} v^i_{pd} l^i_{pd} \tag{7}$$

$g^i_{pd}$ is cost of transportation of residue (unrecovered waste) from processing/diversion facility $p$ to disposal facility $d$ when waste type $i$ is processed. $l^i_{pd}$ is binary (0,1) variable indicating whether residue from processing/diversion facility $p$ can be shifted to disposal facility $d$ when waste type $i$ is processed.

- *Waste processing/handling cost*:

The waste processing/handling cost in the various facilities includes:

Fixed Cost of the Facility + Variable Cost.

(i) Cost incurred (fixed cost and waste handling cost) at the collection centers will be:

$$T_1 = \sum_{j \in J} \left( \vartheta_j + \gamma_j \mu_j \right) \tag{8}$$

$\vartheta_j$ is the fixed cost of waste collection at center $j$ and $\gamma_j$ is per unit waste handling cost at collection center $j$ while $\mu_j$ is the total volumes of wastes at the collection point $j$.

(ii) The cost incurred (fixed cost and waste handling cost) at transfer station facilities will be:

$$\mathrm{T}_2(x) = \sum \left( \beta_t + \sum_{i \in I} \sum_{j \in J} c^i_t x^i_{jt} \tau^i_{jt} \right) \tag{9}$$

$\beta_t$ is the fixed cost of transfer station facility $t$, $c^i_t$ is the unit cost of handling waste type $i$ at transfer station $t$.

(iii) The cost incurred (fixed cost and waste handling cost) at processing/diversion facilities will be:

$$T_3(\bar{x}, \bar{y}) = \sum_{p \in P} \left( \bar{\beta}_p + \sum_{i \in I} \sum_{j \in J} \bar{c}^i_p \bar{x}^i_{jp} \bar{\tau}^i_{jp} + \sum_{i \in I} \sum_{t \in T} \bar{c}^i_p \bar{y}^i_{tp} \bar{h}^i_{tp} \right) \tag{10}$$

$\bar{\beta}_p$ is the fixed cost of processing/diversion facility $p$ and $\bar{c}^i_p$ is the unit cost of handling waste type $i$ at facility $p$.

(iv) The cost incurred (fixed cost and waste handling cost) at the disposal facilities will be:

$$T_4\left(\bar{x}, \tilde{y}, v\right) = \sum_{d \in D}\left(\tilde{\beta}_d + \sum_{i \in I}\sum_{j \in J}\tilde{c}_d^i \tilde{x}_{jd}^i \tilde{\tau}_{jd}^i + \sum_{i \in I}\sum_{t \in T}\tilde{c}_d^i \tilde{y}_{td}^i \tilde{h}_{td}^i + \sum_{i \in I}\sum_{p \in P}\tilde{c}_d^i v_{pd}^i l_{pd}^i\right) \qquad (11)$$

$\tilde{\beta}_d$ is the fixed cost of disposal facility $d$ and $\tilde{c}_d^i$ is the unit cost of handling residue from waste type $i$ at facility $d$.

- *The total cost involved is*:

$$T_{Cost} = F_1\left(x, \bar{x}, \tilde{x}\right) + F_2\left(\bar{y}, \tilde{y}\right) + F_3\left(v\right) + T_1 + T_2\left(x\right) + T_3\left(\bar{x}, \bar{y}\right) + T_4\left(\tilde{x}, \tilde{y}, v\right) \qquad (12)$$

**The Model Objectives**

The objectives of the model are:

(i) To minimize volumes of wastes generated at various waste collection centers. (i.e. to minimize equation (2)).

*(ii)* To minimize the total cost involved in the waste management (i.e. to minimize equation (12)).

**Constraints of the Model**

- *Mass balance Constrains*

(i) All the volumes of wastes at collection center $j$ that are moved to various collection centers should not include recovered/reused material at the collection centers:

$$G_j\left(x, \bar{x}, \tilde{x}\right) = \mu_j\left(1 - \sum_{r \in R}\phi_{rj}\theta_r\right), \qquad \forall j \qquad (13)$$

$\phi_{rj}$ is the percentage of material $r$ recovered ( reused/ recyclable raw material) at collection center $j$. $\theta_r$ is the percentage of material $r$ in the waste.

(ii) All the volumes of wastes moved to facilities in the waste management system should not exceed the amount of wastes in the collection centers.

$$\sum_{j \in J} G_j\left(x, \bar{x}, \tilde{x}\right) \le \sum_{j \in J}\mu_j\left(1 - \sum_{r \in R}\sum_{j \in J}\phi_{rj}\theta_r\right) \qquad (14)$$

- ***Flow Conservation***

The rate of incoming volumes of wastes at any facility must equal the rate of outgoing waste. For the various facilities, this implies that:

(i) Transfer Station Facility:

$$H_t\left(x\right) = \sum_{i \in I}\sum_{p \in P} \bar{y}^i_{tp}\bar{h}^i_{tp} + \sum_{i \in I}\sum_{d \in D} \tilde{y}^i_{td}\tilde{h}^i_{td}, \qquad \forall t \tag{15}$$

(ii) Processing/Diversion Facility:

$$N_p\left(\bar{x},\bar{y}\right) = \sum_{i \in I}\sum_{d \in D} v^i_{pd} l^i_{pd}\ , \qquad \forall p \tag{16}$$

➢ ***Capacity constraints***

Each facility has a single capacity constraint, where capacity is expressed in terms of throughput, i.e. in terms of tons per day sorted, tons per day transferred, tons per day decomposed, tons per day incinerated, tons per day recycled materials, etc. This is also the case for landfill facilities, where the capacity is described in terms of tons of waste treated and deposited per day.

(i) Transfer Stations Capacity:- waste type $i$ moved from various collection centers to transfer station $t$ should not exceed the capacity of transfer station facility:

$$\sum_{i \in I}\sum_{j \in J} x^i_{jt}\tau^i_{jt} \le \Gamma_t\ , \qquad \forall t \tag{17}$$

$\Gamma_t$ is the capacity of transfer station $t$.

(ii) Processing/Diversion Facilities Capacity:- waste type $i$ moved from, various collection centers to processing/diversion facility $p$ should at least not exceed the capacity of the processing/diversion facility.

$$\sum_{i \in I}\sum_{j \in J} \bar{x}^i_{jp}\bar{\tau}^i_{jp} + \sum_{i \in I}\sum_{t \in T} \bar{y}^i_{tp}\bar{h}^i_{tp} \le \Pi_p\ , \qquad \forall p \tag{18}$$

$\Pi_p$ is the capacity of processing/diversion facility $p$.

(iii) Waste Disposal Facilities Capacity:- the amount of unrecovered waste (residue) moved to the disposal center from various facilities in the WMS should at least not exceed the capacity of the disposal center.

$$\sum_{i \in I}\sum_{j \in J} \tilde{x}^i_{jd}\tilde{\tau}^i_{jd} + \sum_{i \in I}\sum_{t \in T} \tilde{y}^i_{sd}\tilde{h}^i_{sd} + \sum_{i \in I}\sum_{p \in P} v^i_{pd} l^i_{pd} \le \Omega_d\ , \qquad \forall d \tag{19}$$

$\Omega_d$ is the capacity of disposal facility $d$.

➢ ***Policy directive***

As matter of policy, certain percentage of the waste at collection centers are to be moved to transfer station and processing facility for separation, recovery of reused/recycle materials, treatment of waste and for easy transportation of the waste to other facility in the waste management system.

(i) Collection centers to transfer station facilities:

$$\sum_{i\in I}\sum_{j\in J}\sum_{t\in T} x_{jt}^{i}\tau_{ji}^{i} \geq \alpha \sum_{j\in J} G_j\left(x,\overline{x},\tilde{x}\right) \tag{20}$$

$\alpha$ is the percentage of waste from collection centers moved to transfer stations.

(ii) Collection Centers to Processing facilities:

$$\sum_{i\in I}\sum_{j\in J}\sum_{t\in T} \overline{x}_{jp}^{i}\overline{\tau}_{jp}^{i} \geq \pi \sum_{j\in J} G_j\left(x,\overline{x},\tilde{x}\right) \tag{21}$$

$\pi$ is the percentage of waste from collection centers moved to processing facilities.

(iii) Transfer Stations to Processing Facilities:

$$\sum_{i\in I} \overline{y}_{tp}^{i}\overline{h}_{tp}^{i} = \eta_{tp} H_t\left(x\right), \qquad \forall t, p \tag{22}$$

$\eta_{tp}$ is the percentage of waste moved from transfer station $t$ to processing facility $p$.

- ***Financial constraint***

Financial constraints require that, for any given period of the waste management (daily, weekly, monthly or annually), the total expenditures should be less than or equal to amount set for the waste management for the period, that is,

$$T_{Cost} \leq B \tag{23}$$

where B is the amount budgeted for waste management for the given period.

- ***Multi-objective Goal Constraints***

The two goals we have in this problem are the goal of minimizing the volumes of wastes at collection centers and goal of minimizing the associated cost. The goal constraint of the multi-objective problem can be sated as follows (deviational variables $s_1^{j-}$ was introduced to equation (13) and, $s_2^-, s_2^+$ were introduced to equation (23)), that is:

$$G_j\left(x,\overline{x},\tilde{x}\right) + s_1^{j-} = \mu_j\left(1 - \sum_{r\in R}\phi_{rj}\theta_r\right), \qquad \forall j \tag{24}$$

$$T_{Cost} + s_2^- - s_2^+ = B \tag{25}$$

$s_1^{j-}$ is the underachievement goal of removing the waste (i.e, amount of waste not removed from the collection center $j$), $s_2^-$ is the underachievement of not using the amount of money budgeted (i.e excess amount of money not utilized) and $s_2^+$ is the overachievement goal of the budgeted amount (i.e, the required additional amount of money to evacuate all the waste).

**Objective Function**

The objective function of multi-objective goal programming problem is:

$$\text{Minimize: } Z = \Delta_1 \sum_{j\in J} \omega_1^{j-} s_1^{j-} + \Delta_2 s_2^{+} \quad (26)$$

$$\text{where, } \sum_{j\in J} \omega_1^{j-} = 1 \quad (27)$$

$\Delta_1$ and $\Delta_2$ are priorities allotted to the two goals. The set of priority structure is usually determined interactively in conjunction with the management. $\omega_1^{j-}$ is the relative weight assigned to the various collection centers, based on relative importance of the collection center during waste evacuation. Analytical Hierarchy Process (AHP) developed by [24] was used to determine the value of the weight $\omega_1^{j-} (j = 1, 2, ..., J)$. $Z$ is the sum of all the deviation from the goals specified by the management. If $Z$ is minimized, all the deviational variables $\left(s_1^{j-}, s_2^{+} \text{ for } j = 1, 2, ..., J\right)$ will be minimized in the order of priorities allotted to them, interactively for each run of the model.

From the foregoing, the complete multi-objective mathematical model is as follows:

$$
\begin{aligned}
&Minimize: \quad Z = \Delta_1 \sum_{j\in J} \omega_1^{j-} s_1^{j-} + \Delta_2 s_2^{+} \\
&Subject \quad to: \\
&G_j(x,\bar{x},\tilde{x}) + s_1^{j-} = \mu_j\left(1 - \sum_{r\in R}\phi_{rj}\theta_r\right), \quad \forall j \\
&\sum_{j\in J} G_j(x,\bar{x},\tilde{x}) = \sum_{j\in J}\mu_j\left(1 - \sum_{r\in R}\phi_{rj}\theta_r\right) \\
&H_t(x) - \sum_{i\in I}\sum_{p\in P}\bar{y}_{tp}^i \bar{h}_{tp}^i + \sum_{i\in I}\sum_{d\in D}\tilde{y}_{td}^i \tilde{h}_{td}^i = 0, \quad \forall t \\
&N_p(\bar{x},\bar{y}) - \sum_{i\in I}\sum_{d\in D} v_{pd}^i l_{pd}^i = 0, \quad \forall p \\
&\sum_{i\in I}\sum_{j\in J} x_{jt}^i \tau_{jt}^i \le \Gamma_t, \quad \forall t \\
&\sum_{i\in I}\sum_{j\in J} \bar{x}_{jp}^i \bar{\tau}_{jp}^i + \sum_{i\in I}\sum_{t\in T}\bar{y}_{tp}^i \bar{h}_{tp}^i \le \Pi_p, \quad \forall p \\
&\sum_{i\in I}\sum_{j\in J} \tilde{x}_{jd}^i \tilde{\tau}_{jd}^i + \sum_{i\in I}\sum_{t\in T}\tilde{y}_{sd}^i \tilde{h}_{sd}^i + \sum_{i\in I}\sum_{p\in P} v_{pd}^i l_{pd}^i \le \Omega_d, \quad \forall d \\
&\sum_{i\in I}\sum_{j\in J}\sum_{t\in T} x_{jt}^i \tau_{jt}^i - \alpha\sum_{j\in J} G_j(x,\bar{x},\tilde{x}) \ge 0 \\
&\sum_{i\in I}\sum_{j\in J}\sum_{p\in P} \bar{x}_{jp}^i \bar{\tau}_{jp}^i - \pi\sum_{j\in J} G_j(x,\bar{x},\tilde{x}) \ge 0 \\
&\sum_{i\in I}\bar{y}_{tp}^i \bar{h}_{tp}^i - \eta_{tp} H_t(x) = 0, \quad \forall t, p \\
&TCost + s_2^{-} - s_2^{+} = B \\
&\sum_{j\in J}\omega_1^{j-} = 1, \quad \omega_1^{j-} > 0 \\
&\tau_{jt}^i, \bar{\tau}_{jp}^i, \tilde{\tau}_{jd}^i, \bar{h}_{tp}^i, \tilde{h}_{td}^i, \tilde{l}_{pd}^i = [0,1], \\
&x_{jt}^i, \bar{x}_{jp}^i, \tilde{x}_{jd}^i, \bar{y}_{tp}^i, \tilde{y}_{td}^i, v_{pd}^i \ge 0, \quad i = 1,2,...,I;\ j = 1,2,...,J;\ t = 1,2,...,T;\ p = 1,2,...,P;\ d = 1,2,...,D \\
&s_1^{j-}, s_2^{-}, s_2^{+} \ge 0, \quad j = 1,2,...,J
\end{aligned}
\quad (28)
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## MODEL APPLICATION

### Study Area

This model has been applied to the waste management system of Abuja, the capital city of Nigeria. The city is located at the geographical center of the country approximately at latitude 9$^o$ 12′ north of the equator and along longitude 7$^o$ 11′ east of the Greenwich Meridian [1]. It has an estimated population of 1.4 million people, of which 405,000 live and work within the municipality [13]. It has a total land area of approximately 713 km$^2$ which is divided into six area councils namely, Abuja Municipal, Abaji, Bwari, Gwagwalada Kuje and Kwali. The climate is generally tropical and it has largely tropical savannah vegetation except for the southern fringes covered by secondary rainforest vegetation. Total annual rainfall in the city averages 1100 mm. The city is located in a scenic valley of rolling grasslands in a relatively undeveloped, ethnically neutral area. Its planners hoped to create a city where none of Nigeria's social and religious groups would be dominant [17 and 4]. The Government institution responsible for solid waste management in the city (Abuja Municipal) is the Abuja Environmental Protection Board (AEPB). The Board's solid waste management portfolio has the following components: City cleaning (concessioned to local contractors in a public private participation arrangement), street sweeping, litter control, solid waste collection, transfer and vegetation control, management of garden, hospital and waste evacuation. Protection and improvement of air, water, land, forest, wildlife and ecological quality, pollution control and environmental health services are also among its mandates [2]. Municipal solid waste management is therefore one of the central mandates of the Board. AEPB solid waste department is responsible for collection, transfer and waste disposal as well as waste material procurements and distribution in the City.

Abuja municipality is divided into 13 waste management operational areas (waste collection area or district). These are: Garki 1,Wuse 1, Wuse2, Central Area, Gwarinpa, Maitama, Asokoro, Jabi, Durimi, Lugbe, Life Camp, Kado and Wuye (see Fig.2). Each of these areas is concessioned to a private sub-contractor in contract arrangement. Within the contract period, all operational responsibility for the given area rests on the sub-contractor while the AEPB assumes a supervisory role. The waste is collected by the contractors at various collection points at the respective area. The waste is not categories or classified at the collection centers or at transfer station. The waste collection is carried out daily for most of the collection area using compacting truck, side loaders, and open tippers; pay loaders, roll-on roll-off trucks etc. There is one recycling facility at Mpape and two transfer station at Kubwa and Gudu. Abuja municipality has two waste disposal site located at Gosa and Ajata, a few kilometer from the city. All the unrecovered waste from the collection areas, transfer stations and recycling facility are taken to one of these disposal sites on daily basis.

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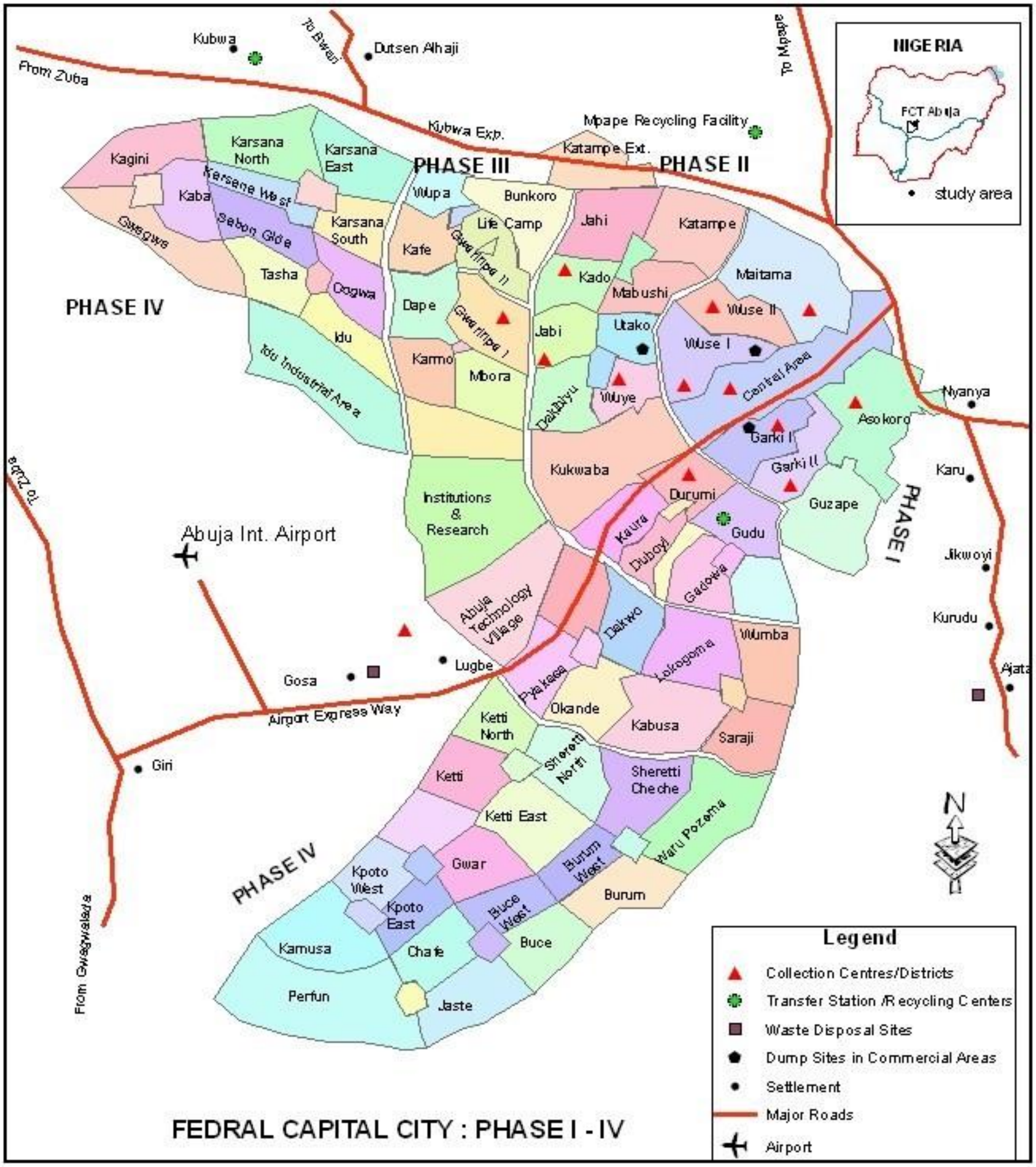


**Fig. 2:** Map of Abuja, Federal Capital City showing the waste collection Centers, Transfer Stations, Recycling Facility and Disposal Sites.

## Data Presentation

The data on Tables 1 to 9 was collected from AEPB, waste management contractors and interview with other solid waste management stake holders.

**Table1:**: Amount of Waste (in Tons) Per Month/Day in the Collection Centers.

| **S. No.** | **Waste Collection Centers (*j*)** | **Amount of Waste/Month** | **Amount of waste/Day** $(\mu_j)$ | **Amount of waste that goes to Waste Management Facilities per Day** $\mu_j\left(1-\sum_{r\in R}\phi_{rj}\theta_r\right)$ |
|---|---|---|---|---|
| **1** | Garki I | 872.36 | 29.80 | **26.93** |
| **2** | Garki II | 1546.28 | 51.55 | **46.59** |
| **3** | Wuse I | 1799.00 | 60.00 | **54.23** |
| 4 | Wuse II | 1952.50 | 65.09 | **58.83** |
| 5 | Central Area | 924.77 | 30.83 | **27.87** |
| 6 | Gwarinpa | 864.87 | 28.83 | **26.06** |
| 7 | Maitama | 1097.00 | 36.57 | **33.05** |
| 8 | Asokoro | 1583.72 | 52.79 | **47.71** |
| 9 | Jabi/Utako/Mboci | 1235.52 | 41.19 | **37.23** |
| 10 | Durimi/Gudo/Apo | 584.07 | 19.47 | **17.60** |
| 11 | Lugbe | 284.55 | 9.46 | **8.55** |
| 12 | Kado | 309.26 | 10.31 | **9.32** |
| 13 | Wuye | 208.08 | 6.94 | **6.27** |
| | Total | | 442.83 | **400.25** |
| Budget Allocation for SWM by AEPB, (**B**) | | ₦154,324,875.57 per month | **₦4,978,221.79 per day** | |

**Source**: Waste Audit Report by Resource Recovery Unit, AEPB, 2012.

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**Table 2**: Transportation Cost of Waste (in ₦/ton) from Collection Centers to Various Waste Facilities in the SWM System.

| S. No. | Waste Collection Center j | Transfer Stations (*t*) | | | | Recycling Facility(*p*) | | Waste Disposal Site (*d*) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Kubwa (*t*=1) | | Gudu (*t*=2) | | Mpape (*p*=1) | | Gossa (*d*=1) | | Ajata (*d*=2) | |
| | | Cost(₦)/20 tons truck | Cost(₦)/ton ($q_{j1}$) | Cost(₦)/20 tons truck | Cost (₦)/ton | Cost(₦)/20 tons truck | Cost (₦)/ton ($\bar{q}_{j1}$) | Cost (₦)/20 tons truck | Cost (₦)/ton | Cost(₦)/20 tons truk | Cost (₦)/ton |
| 1 | Garki I | 40000.00 | 2000.00 | 35000.00 | 1750.00 | 40000.00 | 2000.00 | 55000.00 | 2750.00 | 57000.00 | 2850.00 |
| 2 | Garki II | 38000.00 | 1900.00 | 32000.00 | 1600.00 | 42000.00 | 2100.00 | 48000.00 | 2400.00 | 50000.00 | 2500.00 |
| 3 | Wuse I | 35000.00 | 1750.00 | 37000.00 | 1850.00 | 45000.00 | 2250.00 | 50000.00 | 2500.00 | 55000.00 | 2750.00 |
| 4 | Wuse II | 30000.00 | 1500.00 | 35000.00 | 1750.00 | 33000.00 | 1650.00 | 48000.00 | 2400.00 | 50000.00 | 2500.00 |
| 5 | Central Area | 40000.00 | 2000.00 | 30000.00 | 1500.00 | 35000.00 | 1750.00 | 45000.00 | 2250.00 | 48000.00 | 2400.00 |
| 6 | Gwarinpa | 30000.00 | 1500.00 | 39000.00 | 1950.00 | 34000.00 | 1700.00 | 62000.00 | 3100.00 | 65000.00 | 3250.00 |
| 7 | Maitama | 32000.00 | 1600.00 | 38000.00 | 1900.00 | 38000.00 | 1900.00 | 50000.00 | 2500.00 | 45000.00 | 2250.00 |
| 8 | Asokoro | 40000.00 | 2000.00 | 30000.00 | 1500.00 | 45000.00 | 2250.00 | 48000.00 | 2400.00 | 40000.00 | 2000.00 |
| 9 | Jabi/Utako/Mabuci | 35000.00 | 1750.00 | 40000.00 | 2000.00 | 32000.00 | 1600.00 | 55000.00 | 2750.00 | 57000.00 | 2850.00 |
| 10 | Durimi/Gudu/Apo | 45000.00 | 2250.00 | 20000.00 | 1000.00 | 40000.00 | 2000.00 | 35000.00 | 1750.00 | 55000.00 | 2750.00 |
| 11 | Lugbe | 50000.00 | 2500.00 | 25000.00 | 1250.00 | 45000.00 | 2250.00 | 30000.00 | 1500.00 | 57000.00 | 2850.00 |
| 12 | Kado | 36000.00 | 1800.00 | 32000.00 | 1600.00 | 32000.00 | 1600.00 | 48000.00 | 2400.00 | 50000.00 | 2500.00 |
| 13 | Wuye | 35000.00 | 1750.00 | 39000.00 | 1950.00 | 30000.00 | 1500.00 | 45000.00 | 2250.00 | 48000.00 | 2400.00 |

**Source**: Interview with Waste Management Companies, 2012.

**Table 3 :** Transportation Cost (in ₦/ton) of Waste from Transfer Station to Waste Processing Plant and Disposal Site.

| S/No. | Transfer Station (t) | Waste Processing Plant (p) | | Disposal site, (d) | | | |
|---|---|---|---|---|---|---|---|
| | | | | **Gossa** $(d_1)$ | | **Ajata** $(d_2)$ | |
| | | Cost(₦)/20 tons truck | Cost(₦)/ton ($\bar{b}_{t1}$) | Cost(₦)/20 ton truck | Cost(₦)/ton ($\tilde{b}_{t1}$) | Cost(₦)/20 ton truck | Cost(₦)/ton ($\tilde{b}_{t2}$) |
| 1 | Kubwa $(t_1)$ | 40000.00 | 2000.00 | 60000.00 | 3000.00 | 55000.00 | 2750.00 |
| 2 | Gudu $(t_2)$ | 50000.00 | 2500.00 | 45000.00 | 2250.00 | 50000.00 | 2500.00 |

**Source:** Interview with Waste Management Companies, 2012.

**Table 4:** Transportation Cost per Ton of Waste from Recycling Plant to Waste Disposal Site

| /No. | Processing Plant ($p$) | Disposal site ($d$) | | | |
|---|---|---|---|---|---|
| | | **Gossa** ($d_1$), | | **Ajata** ($d_2$) | |
| | | Cost(₦)/20 ton Truck | Cost(₦)/ ton ($\tilde{g}_{p1}$) | Cost(₦)/20 ton truck | Cost(₦)/ton ($\tilde{g}_{p2}$) |
| 1 | Mpape | 50000.00 | 2500.00 | 50000.00 | 2500.00 |

**Source:** Interview with Waste Management Companies, 2012.

**Table 5**: Fixed Cost, Per Unit Waste Handling Cost and the Capacity of the Disposal Facility

| S/No. | | Transfer Station (t) | | Recycling Facility ($p$) | Disposal Site ($d$) | |
|---|---|---|---|---|---|---|
| | | Kubwa ($t_1$) | Gudu ($t_2$) | Mpape | Gossa ($d_1$) | Ajata ($d_2$) |
| 1 | Fixed Cost (₦)/ton; $\beta_t$, $\bar{\beta}_p$, $\tilde{\beta}_d$ | 50000.00 | 50000.00 | 57600.00 | 50000.00 | 50000.00 |
| 2 | Waste Handling Cost (₦)/ton; $c_t$, $\bar{c}_p$, $\tilde{c}_d$ | 1500.00 | 1500.00 | 1500.00 | 1500.00 | 1500.00 |
| 3 | Capacity of the Facility in tons | 240.00 | 250.00 | 250.00 | 300.00 | 270.00 |

**Source:** Interview with Waste Management Companies and AEPB, 2012.

**Table 6:** Cost, Revenue and Net Benefit from Recovered Material ($r$) in ₦/ton.

| S/No. | Material ($r$) | Cost of Recovered Materials | Market Value of Recovered Materials | Net benefit From Recover Material, ($\psi_r$) |
|---|---|---|---|---|
| 1 | Plastic/ Nylon | 25000.00 | 40000.00 | 15000.00 |
| 2 | Metal/ can | 20000.00 | 38000.00 | 18000.00 |
| 3 | Glass/ Bottles | 50000.00 | 65000.00 | 15000.00 |
| 4 | E-Waste | 80000.00 | 110000.00 | 30000.00 |
| 5 | Paper | 20000.00 | 35000.00 | 15000.00 |
| 6 | Textile | 26000.00 | 35000.00 | 9000.00 |
| 7 | Others | 25000.00 | 35000.00 | 10000.00 |

**Source:** Interview with Waste Recycling Vendors.

**Table7:** Percentage of Material (*r*) in the Solid Waste, and Percentage of Material (*r*) Recovered at Various Facilities.

| S/No. | Material (*r*) | Percentage Volume of Material (*r*) in the Solid waste ($\theta_r$%) | Percentage Volume of Material(*r*) Recovered in Collection. Center ($\phi_{rj}$%) | Percentage Volume of Material (*r*) Recovered in Transfer Station | | Percentage Volume of Material Recycle Facility ($\rho_r$%) |
|---|---|---|---|---|---|---|
| | | | | Kubwa ($\lambda_{r1}$%) | Gudu ($\lambda_{r2}$%) | |
| 1 | Plastic/Nylon | 18.25 | 13.00 | 13.00 | 12.00 | 16.00 |
| 2 | Metal/Can | 3.30 | 15.00 | 14.00 | 13.00 | 17.00 |
| 3 | Glass/Bottles | 2.81 | 12.00 | 12.00 | 12.00 | 14.00 |
| 4 | E- Waste | 1.24 | 16.00 | 13.00 | 12.00 | 14.00 |
| 5 | Paper | 12.56 | 6.00 | 2.00 | 1.00 | 2.00 |
| 6 | Textile | 2.67 | 5.00 | 3.00 | 2.00 | 3.00 |
| 7 | Others | 59.17 | 9.00 | 8.00 | 7.00 | 10.00 |

**Source**: Interview with AEPB and Waste Management Companies.

**Table 8**: Fraction of Recycle/Reuse Material (*r*) Recovered at Collection Centers, Transfer Station and Recycle Facilities.

| S/No. | Material (*r*) | Collection Center | Transfer Station | | Recycle Facility |
|---|---|---|---|---|---|
| | | | Kubwa ($d_1$) | Gudu ($d_2$) | |
| | | ($\theta_r * \phi_{rj}$) | ($\theta_r * \lambda_{r1}$) | ($\theta_r * \lambda_{r2}$) | $\rho_m = (\theta_r * \rho_r)$ |
| **1** | Plastic/ Nylon | 0.0237 | 0.0237 | 0.0219 | 0.0292 |
| 2 | Metal/ Can | 0.0050 | 0.0046 | 0.0043 | 0.0056 |
| 3 | Glass /Bottles | 0.0034 | 0.0034 | 0.0034 | 0.0039 |
| 4 | E- Waste | 0.0020 | 0.0016 | 0.0015 | 0.0017 |
| 5 | Paper | 0.0075 | 0.0025 | 0.0013 | 0.0025 |
| 6 | Textile | 0.0013 | 0.0008 | 0.0005 | 0.0008 |
| 7 | Others | 0.0533 | 0.0473 | 0.0414 | 0.0592 |
| **Total** | | **0.0962** | **0.0840** | **0.0743** | **0.1030** |

**Table 9:** Percentage of Waste Moved from Collection Centers to Various Facilities, and from Transfer Stations to Recycling and Disposal Facilities.

| Waste Management Facilities | Transfer Station | Recycling Facility | Disposal Facility |
|---|---|---|---|
| **Collection Centers** | 50 | 30 | 20 |
| **Transfer Station** | – | 70 | |

**Source**: Interview with AEPB.

**Priority Weights**

Three criteria were used to evacuate the waste in the various collection centers. These are: Population Density (PD), Very Important Personalities (VP) and Road Network / Drainage Channels (RNDC). Using Analytical Hierarchy Process (AHP), pair wise comparison of these criteria was determined as shown in Tables 10a and 10b. In a similar manner, pair wise comparison of preferences for the collection of waste at the various collection centers in relation to each of the criteria PD, VP, and RNDC was also determined and the weights $\left(w_1^{j-}\right)$ for the waste collection centers calculated as shown in Table 11.

**Table10a**: The Pair wise Comparison Matrix for three Criteria.

| | PD | VP | RNDC |
|---|---|---|---|
| PD | 1 | 5 | 2 |
| VP | ⅕ | 1 | 3 |
| RNDC | ½ | ⅓ | 1 |

**Table10b**: The syntheses Matrix for the Three Criteria

| | PD | VP | RNDC | **Priority** |
|---|---|---|---|---|
| PD | 0.5882 | 0.7895 | 0.3333 | **0.5703** |
| VP | 0.1176 | 0.1579 | 0.5000 | **0.2585** |
| RNDC | 0.2941 | 0.0526 | 0.1667 | **0.1711** |
| | | | Total | **1.000** |

**Table 11**: Overall AHP Priority Weights for the waste collection centers

| | Criterion Ranking | | | |
|---|---|---|---|---|
| **Waste collection Centers** | **PD (0.5703)** | **VP (0.2585)** | **RNDC (0.1711)** | **Weight** $\left(w_1^{j-}\right)$ |
| Garki I | 0.0715 | 0.0242 | 0.0271 | **0.1228** |
| Garki II | 0.0642 | 0.0136 | 0.0141 | **0.0919** |
| Wuse I | 0.1073 | 0.0142 | 0.0161 | **0.1376** |
| Wuse II | 0.0788 | 0.0193 | 0.0158 | **0.1140** |
| Cent. A. | 0.0228 | 0.0296 | 0.0239 | **0.0763** |
| Gwarinpa | 0.0382 | 0.0111 | 0.0191 | **0.0684** |
| Maitama | 0.0127 | 0.0510 | 0.0066 | **0.0702** |
| Asokoro | 0.0145 | 0.0419 | 0.0086 | **0.0650** |
| Jabi | 0.0402 | 0.0149 | 0.0071 | **0.0622** |
| Durimi | 0.0360 | 0.0081 | 0.0114 | **0.0555** |
| Lugbe | 0.0373 | 0.0144 | 0.0090 | **0.0606** |
| Kado | 0.0232 | 0.0101 | 0.0082 | **0.0415** |
| Wuye | 0.0240 | 0.0062 | 0.0043 | **0.0346** |

## Results and Discussion

The above multi-objective linear programming model is a weighted preemptive goal programming model in which the major goals of the problem is to minimize the waste in the collection centers, which is the primary objective (first priority goal) and minimize the associated cost of evacuating the waste in the various collection centers, which is the secondary objective(second priority goal). A spreadsheet model of the problem was formulated and Microsoft Excel Solver 14.0 was used to solve the problem. The first priority goal was grouped, weighted and solved. Thereafter, the second priority goal was formulated and solved. Tables 12 and 13 shows the solution obtained after the second priority goal was solved. The final value of the objective function is zero as shown in Table 12, which indicates that both goals are perfectly satisfied. That is, all the waste in the collection centers are evacuated with minimum cost.

Table13 shows the values of the various decision variables. The values of decision variables $x_{j2}, j = 2, 4, 5, 7, 8, 12, 13$ indicates that, 46.59, 58.83, 27.89, 33.05, 18.19, 9.32, 6.27 tons of waste from collection centers 2, 4, 5, 7,8,12 and 13 should be moved to transfer station two ( Gudu). No waste should be moved to transfer station one (Kubwa) from the collection centers, this is indicated by the values of decision Variables $x_{j1} = 0,\ j = 1, 2, ..., 13.$ The values of the decision variables, $\bar{x}_{j1}, j = 1, 3, 8, 10, 11$ indicates that, 10.17, 54.23, 29.52, 17.60, 8.55 tons of waste should be moved from collection centers 1, 3, 8, 10, and 11 to recycling plant at Mpape. The values of the decision variables $\tilde{x}_{j2}, j = 1, 6, 9$ indicate that, 16.72, 26.06, 37.23 tons of waste should be moved from collection centers 1, 6 and 9 to disposal center two (Ajata). The value of the decision variable $\bar{y}_{21}$ indicates that, 129.68 tons of waste should be moved from transfer station two (Gudu) to recycling plant at Mpape. The values of the decision variables $\tilde{y}_{22}$ and $v_{12}$ indicate that 55.58 and 134.37 tons of waste should be moved from transfer station two (Gudu) and recycling plant at Mpape to disposal center two (Ajata). The value of the decision variable $v_{11}$ indicate that 89.67 tons of waste should be moved from recycling plant at Mpape to disposal center one (Gossa). The values of the deviation variables $s_1^{j-} = 0,\ j = 1, 2, ..., 13$ indicate that no waste is left in the collection centers. The deviation variable $s_2^- = 1644696.00$ as shown on Table 13. This indicates that the sum of ₦1, 644,696.00 is lost daily to contractors, which further implies that a daily budget provision of ₦3, 333,525.79, instead of N4,978,221.79 is adequate for waste evacuation from all collection centers.

**Table 12**: Objective Function

| **Variable Name** | **Original Value** | **Final Value** |
|---|---|---|
| Deviation Variables | 0.00 | 0.00 |
| | | |

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**Table 13**: Values of The Decision Variable

| S/No . | Decision Variable | Original Value | Final Value |
|---|---|---|---|
| 1 | $x_{11}$ | 0.00 | 0.00 |
| 2 | $x_{21}$ | 0.00 | 0.00 |
| 3 | $x_{31}$ | 0.00 | 0.00 |
| 4 | $x_{41}$ | 0.00 | 0.00 |
| 5 | $x_{51}$ | 0.00 | 0.00 |
| 6 | $x_{61}$ | 0.00 | 0.00 |
| 7 | $x_{71}$ | 0.00 | 0.00 |
| 8 | $x_{81}$ | 0.00 | 0.00 |
| 9 | $x_{91}$ | 0.00 | 0.00 |
| 10 | $x_{10,1}$ | 0.00 | 0.00 |
| 11 | $x_{11,1}$ | 0.00 | 0.00 |
| 12 | $x_{12,1}$ | 0.00 | 0.00 |
| 13 | $x_{13,1}$ | 0.00 | 0.00 |
| 14 | $x_{12}$ | 0.00 | 0.00 |
| 15 | $x_{22}$ | 0.00 | 46.59 |
| 16 | $x_{32}$ | 0.00 | 0.00 |
| 17 | $x_{42}$ | 0.00 | 58.83 |
| 18 | $x_{52}$ | 0.00 | 27.87 |
| 19 | $x_{62}$ | 0.00 | 0.00 |
| 20 | $x_{72}$ | 0.00 | 33.05 |
| 21 | $x_{82}$ | 0.00 | 18.19 |
| 22 | $x_{92}$ | 0.00 | 0.00 |
| 23 | $x_{10,2}$ | 0.00 | 0.00 |
| 24 | $x_{11,2}$ | 0.00 | 0.00 |
| 25 | $x_{12,2}$ | 0.00 | 9.32 |
| 26 | $x_{13,2}$ | 0.00 | 6.27 |
| 27 | $\bar{x}_{11}$ | 0.00 | 10.17 |
| 28 | $\bar{x}_{21}$ | 0.00 | 0.00 |
| 29 | $\bar{x}_{31}$ | 0.00 | 54.23 |
| 30 | $\bar{x}_{41}$ | 0.00 | 0.00 |
| 31 | $\bar{x}_{51}$ | 0.00 | 0.00 |
| 32 | $\bar{x}_{61}$ | 0.00 | 0.00 |
| 33 | $\bar{x}_{71}$ | 0.00 | 0.00 |
| 34 | $\bar{x}_{81}$ | 0.00 | 29.52 |
| 35 | $\bar{x}_{91}$ | 0.00 | 0.00 |
| 36 | $\bar{x}_{10,1}$ | 0.00 | 17.60 |
| 37 | $\bar{x}_{11,1}$ | 0.00 | 8.55 |
| 38 | $\bar{x}_{12,1}$ | 0.00 | 0.00 |
| 39 | $\bar{x}_{13,1}$ | 0.00 | 0.00 |
| 40 | $\tilde{x}_{11}$ | 0.00 | 0.00 |
| 41 | $\tilde{x}_{21}$ | 0.00 | 0.00 |
| 42 | $\tilde{x}_{31}$ | 0.00 | 0.00 |
| 43 | $\tilde{x}_{41}$ | 0.00 | 0.00 |
| 44 | $\tilde{x}_{51}$ | 0.00 | 0.00 |
| 45 | $\tilde{x}_{61}$ | 0.00 | 0.00 |
| 46 | $\tilde{x}_{71}$ | 0.00 | 0.00 |
| 47 | $\tilde{x}_{81}$ | 0.00 | 0.00 |
| 48 | $\tilde{x}_{91}$ | 0.00 | 0.00 |
| 49 | $\tilde{x}_{10,1}$ | 0.00 | 0.00 |
| 50 | $\tilde{x}_{11,1}$ | 0.00 | 0.00 |
| 51 | $\tilde{x}_{12,1}$ | 0.00 | 0.00 |
| 52 | $\tilde{x}_{13,1}$ | 0.00 | 0.00 |
| 53 | $\tilde{x}_{12}$ | 0.00 | 16.72 |
| 54 | $\tilde{x}_{22}$ | 0.00 | 0.00 |
| 55 | $\tilde{x}_{32}$ | 0.00 | 0.00 |
| 56 | $\tilde{x}_{42}$ | 0.00 | 0.00 |
| 57 | $\tilde{x}_{52}$ | 0.00 | 0.00 |
| 58 | $\tilde{x}_{62}$ | 0.00 | 26.06 |
| 59 | $\tilde{x}_{72}$ | 0.00 | 0.00 |
| 60 | $\tilde{x}_{82}$ | 0.00 | 0.00 |
| 61 | $\tilde{x}_{92}$ | 0.00 | 37.23 |
| 62 | $\tilde{x}_{10,2}$ | 0.00 | 0.00 |
| 63 | $\tilde{x}_{11,2}$ | 0.00 | 0.00 |
| 64 | $\tilde{x}_{12,2}$ | 0.00 | 0.00 |
| 65 | $\tilde{x}_{13,2}$ | 0.00 | 0.00 |
| 66 | $\bar{y}_{11}$ | 0.00 | 0.00 |
| 67 | $\bar{y}_{21}$ | 0.00 | 129.68 |
| 68 | $\tilde{y}_{11}$ | 0.00 | 0.00 |
| 69 | $\tilde{y}_{21}$ | 0.00 | 0.00 |
| 70 | $\tilde{y}_{12}$ | 0.00 | 0.00 |
| 71 | $\tilde{y}_{22}$ | 0.00 | 55.58 |
| 72 | $v_{11}$ | 0.00 | 89.67 |
| 73 | $v_{12}$ | 0.00 | 134.37 |
| 74 | $s_1^{1-}$ | 0.00 | 0.00 |
| 75 | $s_1^{2-}$ | 0.00 | 0.00 |
| 76 | $s_1^{3-}$ | 0.00 | 0.00 |
| 77 | $s_1^{4-}$ | 0.00 | 0.00 |
| 78 | $s_1^{5-}$ | 0.00 | 0.00 |
| 79 | $s_1^{6-}$ | 0.00 | 0.00 |
| 80 | $s_1^{7-}$ | 0.00 | 0.00 |
| 81 | $s_1^{8-}$ | 0.00 | 0.00 |
| 82 | $s_1^{9-}$ | 0.00 | 0.00 |
| 83 | $s_1^{10-}$ | 0.00 | 0.00 |
| 84 | $s_1^{11-}$ | 0.00 | 0.00 |
| 85 | $s_1^{12-}$ | 0.00 | 0.00 |
| 86 | $s_1^{13-}$ | 0.00 | 0.00 |
| 87 | $s_2^{+}$ | 0.00 | 0.00 |
| 88 | $s_2^{-}$ | 0.00 | 1644696.000 |

## CONCLUSION

Minimization of volume of wastes and resources available to evacuate them are always in conflict. Huge budgetary allocations are provided to evacuate volumes of waste. The multi-objective mathematical model developed in this study is designed to solve the problem of conflicting objectives of minimization of volumes of wastes in various collection centers considering their comparable importance in the municipality and associated cost. An application of the model was demonstrated through a case study of Abuja Municipal Council, Federal Capital of Nigeria. The study revealed that the amount allocated daily by AEPB for evacuation of volumes of waste in the municipality is in excess. It has been shown that 33% (₦1, 644,696.00) of the amount allocated daily is lost to contracted waste management companies after evacuation of the wastes. This amount ought to have been savings for the solid waste management agency. The proposed model provides operational and financial information, on solid waste evacuation and contributes effectively to decision making process of solid waste management in our municipal councils.

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